Diffraction of a Shock Wave by a Compression Corner: Part II – Single Mach Reflection

Vijaya Shankar,* Paul Kutler,† and Dale Anderson‡
Iowa State University, Ames, Iowa
and
NASA Ames Research Center, Moffett Field, Calif.

Theme

THE problem of shock wave diffraction, that is, the deflection of a shock whose normal path has been impeded by some obstacle, is of interest to those investigating the nuclear blast fields around aerospace vehicles and flush-mounted surface structures. The simplest laboratory experiment designed to study the shock diffraction problem consists of a two-dimensional wedge or ramp mounted on the wall of a shock tube. Depending on the angle of inclination of the ramp with respect to the shock tube wall θ_r and the strength of the planar shock (with Mach number M_s), either regular reflection or one of several types of Mach reflection occurs.[1] Regardless of the type of reflection process, the shock-diffraction problem is self-similar with respect to time since there is no characteristic length associated with the problem.

The purpose of this paper is to develop a numerical procedure for solving the single Mach reflection case which possesses a smooth reflected shock. The shock-fitting program developed to solve the regular reflection case[2] cannot handle single Mach reflection because of the presence of added complexities such as the Mach stem and the slip surface.

Contents

A typical single Mach reflection is shown in Fig. 1a. The self-similar flowfield is somewhat complicated in this case by the existence of a triple point at which the reflected shock, the Mach stem, and the incident shock coincide. Emanating from the triple point is a slip surface which intersects the ramp at the vortical singularity. A sonic line exists in most of the single Mach reflection cases in the region between the reflected shock and the slip surface. Below this sonic line (region I) and in the region between the Mach stem and the slip line (region III), the self-similar Mach number is subsonic, while above the sonic line (region II), it is supersonic.

In this problem, there are two self-similar stagnation points, that is, points at which the self-similar velocity components are zero; the first is located at the juncture of the wall and the ramp (a saddle point of streamlines), and the second, termed a vortical singularity, is located at the point where the slip surface intersects the ramp (a nodal point of streamlines). All the self-similar streamlines converge at the vortical singularity, and thus the entropy is multivalued. The value of entropy on the stagnation streamline and along the ramp up to the vortical singularity is equal to that behind the normal part of the reflected shock, while the entropy between the vortical singularity and the Mach stem is the same as that behind the foot of the Mach stem.

A Cartesian coordinate system is used in the problem formulation with the origin located at the juncture of the wall and the ramp. The x axis is aligned with the wall and the y axis is normal to the wall. In order to treat the reflected shock and the Mach stem as outer boundaries of the computational domain, a double normalizing transformation of the form

$$\tau = t \qquad \eta = \frac{x - x_b(\xi,\tau)}{x_s(\xi,\tau) - x_b(\xi,\tau)} \qquad \xi = \frac{y}{y_s(\eta,\tau)} \tag{1}$$

is incorporated. In Eq. (1), $x_s(\xi,\tau)$ represents the reflected shock, $x_b(\xi,\tau)$ the ramp, and $y_s(\eta,\tau)$ the Mach stem. The governing equations (continuity, x and y momentum, and energy) are rearranged in strong conservation-law form to yield

$$(U/J)_\tau + [(U\eta_t + E\eta_x + F\eta_y)/J]_\eta + [(U\xi_t + E\xi_x + F\xi_y)/J]_\xi = 0 \tag{2}$$

where U, E, and F are the Cartesian conservative vectors which are functions of the pressure p, the density ρ, the total energy per unit volume e, and the velocity components u and v. In Eq. (2), J is the Jacobian of the transformation and η_x, η_y, η_t, ξ_x, ξ_y, and ξ_t are the geometric derivatives. The pressure, density, and velocity components are related to the energy for an ideal gas by the following equation:

$$e = p/(\gamma - 1) + \rho(u^2 + v^2)/2 \tag{3}$$

Because of the self-similar property of the flow, the term $(U)_\tau$ in Eq. (2) approaches zero as τ gets large, thus establishing a convergence criterion.

The transformation given by Eq. (1) results in the computational plane shown in Fig. 1b. It is bounded by the reflected shock and the Mach stem, both of which are per-

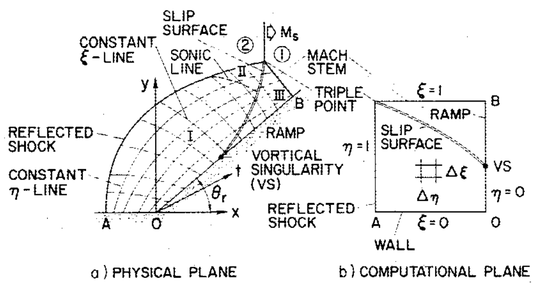

Fig. 1 Coordinate system and computational plane.

Presented as Paper 77-89 at the AIAA 15th Aerospace Sciences Meeting, Los Angeles, Calif., Jan. 24-26, 1977; submitted Jan. 31, 1977; synoptic received July 26, 1977. Full paper available from AIAA Library, 750 Third Avenue, New York, N.Y. 10017. Price: Microfiche, $2.00; hard copy, $5.00. **Order must be accompanied by remittance.**

Index categories: Supersonic and Hypersonic Flow; Shock Waves and Detonations; LV/M Aerodynamics.

*Research Assistant, Iowa State University stationed at NASA Ames Research Center. Presently Member Technical Staff, Science Center, Rockwell International, Thousand Oaks, Calif. Student Member AIAA.

†Research Scientist, Computational Fluid Dynamics Branch, Ames Research Center, NASA. Associate Fellow AIAA.

‡Professor, Department of Aerospace Engineering, Iowa State University. Member AIAA.

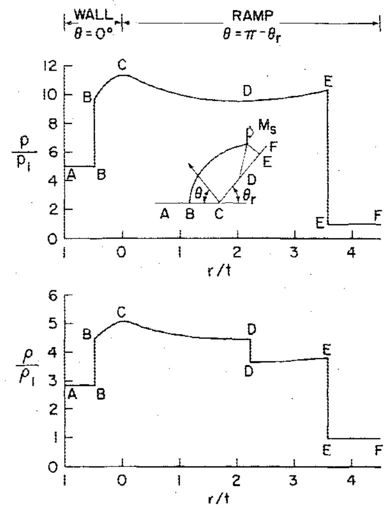

Fig. 2 Surface pressure and density distribution along the wall and the ramp; $M_s = 2.1, \theta_r = 40$ deg.

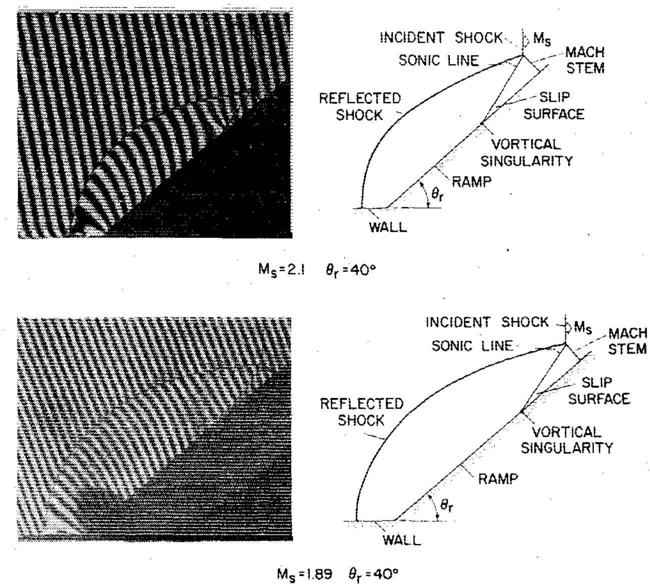

Fig. 3 Comparison of experimental and computed wave structure; $M_s = 1.89, \theta_r = 40$ deg; $M_s = 2.1, \theta_r = 40$ deg.

meable boundaries, and by the wall and the ramp, both of which are impermeable boundaries. The slip surface discontinuity "floats" within the mesh generated by the double normalization (Fig. 1a).

For a given incident shock Mach number M_s and the ramp angle θ_r, initial conditions are specified at each mesh point at time $\tau = 1$. The details are given in Ref. 1.

The second-order accurate predictor-corrector scheme devised by MacCormack[3] is used to integrate Eq. (2). The position, shape, and speed of the reflected shock and Mach stem are determined at each step of the time-asymptotic integration procedure by employing the unsteady version of Thomas' "pressure approach."[4] In this approach, it is necessary to know only the pressure behind the shock in order to alter its position for the next time level. This required pressure is obtained from the finite-difference algorithm.

At the ramp and the wall a simple Euler predictor/modified Euler corrector with one-sided ξ derivatives at the wall and η derivatives at the ramp for Eq. (2) is used. The tangency condition $v = 0$ at the wall and $v = u \tan\theta_r$ at the ramp is imposed after the corrector step. Knowing the location of the vortical singularity, appropriate entropy levels are assigned to the surface grid points.

The slip surface, which floats within the (η,ξ) mesh system, is fitted using Moretti's[5] concept of floating-fitting. In this approach, differencing across the discontinuity is strictly forbidden, and special one-sided differences are used at grid points neighboring the slip surface. A local method of characteristics is employed to compute the flow conditions along both sides of the slip surface.

The computational grid for a typical, single Mach reflection case consisted of 11 points in the η-direction and 31 points in the ξ direction. An average of 400 iterations was required to obtain a converged solution and used approximately 15 min of computer time on an IBM 360/67.

Law and Glass[6] performed a series of experiments on the shock diffraction problem for various gases recording the results using a Mach-Zehnder interferometer. Numerical results were generated for two of these cases ($M_s = 1.89$, $\theta_r = 40$ deg; $M_s = 2.1$, $\theta_r = 40$ deg) which resulted in single Mach reflection. These results are presented in Figs. 2 and 3.

The density and pressure distributions along the wall and the ramp for an incident shock Mach number of 2.1 and a ramp angle of 40 deg are shown in Fig. 2. The juncture of the wall and the ramp is a stagnation point (point C) at which pressure and density reach a local maximum. At the vortical singularity (point D), the pressure is continuous and reaches a local minimum.

A comparison of the experimental interferogram with the numerically computed wave structure is shown in Fig. 3. The slip surface in the numerical solution is nearly straight, as can be observed in the experiment. In addition, a small, self-similar, supersonic region lies between the slip surface and the reflected shock. The sonic line bounding this supersonic region is shown in the numerical results. The triple point trajectory angle χ in the numerical solution is larger than that shown in the experimental interferogram. The reason for the discrepancy is probably twofold: First, the viscous effects (the majority of which can be observed near the wall-ramp intersection) might have the effect of decreasing the ramp angle as a result of the boundary-layer growth with distance from the Mach stem foot. The reduced ramp angle in turn results in a larger triple point trajectory angle. Second, the computed solution assumes flow of an ideal gas ($\gamma = 1.4$). Thus, high-temperature effects on the internal energy, such as molecular, vibrational excitations, are not taken into account.

References

[1] Shankar, V., Kutler, P., and Anderson, D.A., "Diffraction of a Shock Wave by a Compression Corner, Part II – Single Mach Reflection," AIAA Paper 77-89, Los Angeles, Calif., Jan. 1977.

[2] Kutler, P. and Shankar, V., "Diffraction of a Shock Wave by a Compression Corner: Part I – Regular Reflection," *AIAA Journal*, Vol. 15, Feb. 1977, pp. 197-203.

[3] MacCormack, R.W., "The Effect of Viscosity in Hypervelocity Impact Cratering," AIAA Paper 69-354, 1969.

[4] Thomas, P.D., Vinokur, M., Bastianon, R., and Conti, R.J., "Numerical Solution for the Three Dimensional Inviscid Supersonic Flow of a Blunt Delta Body," *AIAA Journal*, Vol. 10, July 1974, pp. 887-894.

[5] Moretti, G., "Experiments in Multidimensional Floating Shock-Fitting," PIBAL Report No. 73-18, Polytechnic Institute of Brooklyn, Aug. 1973.

[6] Law, C.K. and Glass, I.I., "Diffraction of Strong Shock Waves by a Sharp Compressive Corner," Canadian Aeronautics and Space Institute Transaction, Vol. 4, No. 1, pp. 2-12, March 1971.